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VISCOUS THEORY OF ANGULAR FOLDING BY FLEXURAL FLOW

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ABSTRACT

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A continuous variation in fold style from chevron to very rounded profiles can be generated in an infinite viscous multilayer, initially gently buckled, under different conditions of applied stress. A well-foliated medium, considered in two dimensions only, is assumed to deform by flexural flow to give similar folds. At rest it has a single overall Newtonian viscosity; under differential stress this viscosity at any point is proportional to the compressive stress normal to the shear planes at that point. For the upright symmetrical folds considered, the results show that angular folds develop when the ratio of horizontal to vertical stress components is large, whereas rounded profiles result when this ratio approaches unity. Thus fold styles, together with depth of burial estimates from the degree of metamorphism, can be used to estimate horizontal stress magnitudes, for instance in an orogenic belt. The time duration for the formation of a given fold can also be deduced if the overall viscosity can be estimated.

INTRODUCTION

The theory proposed in this paper concerns the style of folds developed by flexural flow in an initially gently buckled well-bedded or well-foliated medium, under the action of differential stresses. The aim is to show that angular or chevron similar folds can be produced by a simple viscous model, and that these folds grade into more rounded styles under different physical conditions.

The mechanism of flow within layers envisaged here has been designated "flexural flow" by Donath and Parker (1964), and is characterised by movement of material away from or towards fold hinges, usually resulting in relative thickening at the hinges. A "flexural slip" mechanism implies a maintenance of constant layer thickness and a decrease in radius of curvature towards the concave side of a fold, i.e., concentric folding, whereas flexural flow commonly results in an approach of fold geometry to that of a similar fold, by which fold style in profile is maintained along the axial plane. The limiting case of these two geometries is the angular fold, in which constancy of layer thickness (except at the fold hinge) and fold style is preserved. In the model developed below the flexural flow mechanism is assumed to give rise to an ideal similar fold geometry, as in the buckling treatments by Biot (1964c; 1965c, p.254), but since the emphasis is on the more angular styles of folding the effect of increase in layer thickness towards the hinge is neglected in the analysis.

It is therefore assumed that the only type of strain during the folding process is a shearing parallel to the layering, which requires that the viscosity coefficient for shear strain in this direction be much less than for shear strain in any other direction. So the theory must be limited to well-layered media, for which this condition is likely to hold. Angular folds in nature do, in fact, tend to occur only in well-bedded or well-foliated rocks.

As with most previous treatments, notably by Biot (1961, 1964a, b, 1965a, b) and Ramberg (1961, 1963a, 1964) a Newtonian viscosity is assumed, but in contrast to the work of these authors the effect of individual layer thickness is not considered, being taken to be small in comparison with the wavelength of the folding. The theory of an extended sequence of alternating layers folded similarly has been developed by Bayly (1964); angular folds result because gentle straight-limbed folds are used as the starting point for the calculation. He concludes that the assumption of viscosity independent of stress is probably unjustified. Chapple (1968) also casts doubt on this assumption, observing that the folds produced by his theoretical model (and by others) are too broad, especially at high amplitudes.

The simple relationship between viscosity and stress chosen here is that viscosity is directly proportional to the compressive stress normal to the planes undergoing shear strain. Fluid theory (e.g., Frenkel, 1946) predicts that viscosity should increase exponentially with hydrostatic pressure, but that this law does not strictly hold under high pressures, whereas published data (e.g. Kaye and Laby, 1966) on the variation of viscosity of organic liquids under confining pressures from 1–12 kbar suggest a relationship lying somewhere between a proportional and an exponential increase. For simplicity, the former alternative is used in the theory. Rocks may well exhibit a viscosity which decreases with increasing shear stress (H. Ramberg, personal communication, 1971. See also the discussion by Ramberg, 1961, p.389). A theoretical model in which, in addition to the previous conditions, viscosity is inversely proportional to shearing stress is discussed below, but since there is at present little experimental evidence concerning the relationship between viscosity and shear stress, the detailed analysis is confined to the model in which viscosity remains independent of shear stress.

Under static conditions, at a given temperature and pressure, the rock will have an overall constant viscosity, assuming either that there is little viscosity contrast between the layers, or that the layering is too fine relative to the volume of rock considered to allow resolution into separate layer viscosities. Under the dynamic conditions of folding this viscosity will be increased by different amounts over the arc of a fold, depending upon the compressive stress normal to the shear planes at that point.

The assumption that viscosity is proportional to normal compressive stress allows a variation in fold style which would not be possible using a simple viscous fold model, and substitution of an exponential law for the proportional relationship chosen would not alter the results qualitatively, but only exaggerate the importance of differential stress in producing angular styles.

THEORY

The viscous model

In a fluid at rest the hydrostatic or confining pressure ρ is the normal compressive stress σ acting across any plane within it. When the fluid is in motion, adjacent parallel elementary planes undergo shear strain relative to one another due to the action of external stresses. These stresses can be resolved into a shear stress τ parallel to the planes and a compressive stress σ_n normal to the planes. Thus for any element the dynamic equivalent of the scalar hydrostatic pressure ρ is the normal compressive stress σ_n. Because it has been shown that a reasonably simple description of the behaviour of viscosity is an increase proportional to ρ, it follows that in the dynamic case the viscosity at a particular point will be proportional to σ_n, the normal compressive stress at that point.

Consider such an elementary unit, for instance a pair of adjacent bedding or foliation planes of length Δl, under the influence of external stresses which can be resolved into three orthogonal principal compressive stresses σ_1, σ_2 and σ_3, (Fig.1A). For simplicity the

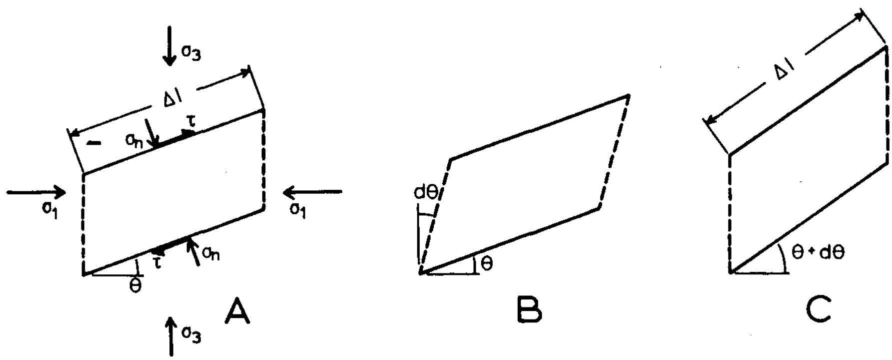

Fig.1. A. External stresses σ_1 and σ_3 acting on an element of bedding of length Δl, dipping at an angle θ. σ_1 and σ_3 can be resolved into a normal compressive stress σ_n and a shear stress τ. B. τ causes the parallelogram of A to shear through a small angle $d\theta$ in a small time dt. C. The dotted vertical boundaries of the element remain vertical during similar folding, so the shear angle $d\theta$ is an addition to the dip θ.

maximum stress σ_1 is taken as horizontal and the minimum stress σ_3 as vertical. The element is one of a chain of such elements forming the arc of a gentle fold buckled under the action of external stresses σ_1, σ_2 and σ_3, the axis of the fold being parallel to σ_2 (Fig.2). σ_2 is assumed to remain constant, so the element can be considered in two dimensions only, parallel to the $\sigma_1 - \sigma_3$ plane, and σ_2 can be neglected. The shearing stress τ and the normal compressive stress σ_n on the layering are the resultants of σ_1 and σ_3 only, and also lie in this plane. As in this case the mechanism of flexural flow between adjacent planes is assumed to generate folds of "similar" type, isogons at any stage are parallel to the σ_3

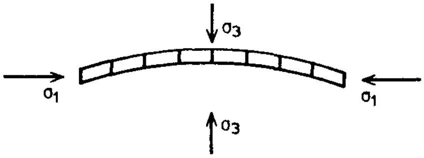

Fig.2. Chain of elements as in Fig.1 forming an arc of a fold buckled under external stresses σ_1, σ_2, and σ_3. In fold profile the intermediate compressive stress σ_2 can be neglected.

direction. The imaginary divisions between the element of Fig.1A and its neighbours are therefore taken as vertical initially; however, they have no physical expression as planes of movement.

Under the action of τ the parallelogram of Fig.1A is deformed with time to that of Fig.1B; but the angle $d\theta$ through which it has sheared must be regarded as an addition to the initial dip θ, because the boundaries with the adjoining elements remain vertical (Fig.1C) – by definition, as this condition is intrinsic to the development of similar folds.

Because the model is a Newtonian fluid, the rate of shear strain $d\theta/dt$, which is also the rate of increase of dip, is proportional to τ, or:

$$d\theta/dt = \tau/\eta \tag{1}$$

where η is the viscosity. But η is proportional to σ_n, so:

$$\eta = \sigma_n/c \tag{2}$$

where c is a constant.

$$\therefore d\theta/dt = c\tau/\sigma_n \tag{3}$$

To find the final dip of the element of layering after it has been acted upon for a given time T under various magnitudes of σ_1 and σ_3 (each assumed to be constant during the deformation), eq.3 is integrated with respect to time t. The result (see Appendix) is:

$$\left[\frac{\sigma_1 + \sigma_3}{\sigma_1 - \sigma_3} \cdot \log|\tan\theta| - \log|\sin 2\theta|\right]_{\theta_I}^{\theta_F} = 2cT \tag{4}$$

where θ_F is the final dip of the element, initially at θ_I, after a given time T. Note that the constant c must have dimension [time^{-1}] in order to satisfy eq.4. Inspection of eq.2, in which c is defined, confirms this.

Application

The simplest way to solve eq.4 for θ is graphically. It can be written:

$$f(\theta)_{\theta_F} - f(\theta)_{\theta_I} = 2cT \tag{5}$$

A family of graphs of:

$$f(\theta) = \frac{\sigma_1 + \sigma_3}{\sigma_1 - \sigma_3} \log_{10} |\tan \theta| - \log_{10}|\sin 2\theta|$$

is drawn against θ for several values of the constant $\frac{\sigma_1 + \sigma_3}{\sigma_1 - \sigma_3} = R$ (Fig.3). For example, if σ_1 is three times bigger than σ_3 then $R = 2$. An element having a dip θ_I defines the value of the function $f(\theta)_{\theta_I}$ on the curve for $R = 2$ (the point X in Fig.3). Eq.5 can be written:

$$f(\theta)_{\theta_F} = f(\theta)_{\theta_I} + K \tag{6}$$

where K is a new constant equal to $2cT$ and therefore dimensionless, but proportional to the total time T of the process. K is added to the value of the function at X, as in eq.6, giving the point Y from which the point Z on the same curve ($R = 2$) fixes the final dip θ_F.

Thus for a series of elements defining an initial fold the shape of the resultant fold at any later time can be constructed by reading off individually the later dips of the elements, after addition of the same constant K, and joining them up again into a smooth curve. The result will of course be an approximation, but as long as the starting curve is divided up into a sufficiently large number of segments the inaccuracy in shape of the reconstructed curve will be small.

In each of the four sets of constructions for the four values of R chosen, the starting curve ($K = 0$) is a circular arc a quarter of a wavelength long, having a maximum dip on its flank of 5° (Fig.4). It is divided up into ten equal segments dipping at 5°, 4½°, ... 1°, ½° respectively. For comparison of style the reconstructions are computed for the same "maximum dips" on the flank, viz. 20°, 40°, and 60°, rather than for equal time durations (i.e., equal values of K). De Sitter (1964) and Ramsay (1967) have shown from geometrical considerations that angular folds "lock up" at limb-dips of around 60°; if so, strain by flexural flow will cease and probably be replaced by homogeneous strain or by flow parallel to a developing axial plane cleavage. The maximum dip computed at the inflection point is therefore taken as 60°.

Results

Some of the quarter-wave curves of Fig.4 are extended to a complete wavelength or more in Fig.5. The resulting folds indicate that for the same limb-dip of 60° (Fig.5A) angular folds develop when R tends towards unity, whereas rounded styles form for large values of R. In the former case, in which $R = 1.1$, σ_1 is about ten times greater than σ_3, whereas for $R = 10$, σ_1 is only slightly greater than σ_3. In addition, the time factor for the very rounded fold is about seventeen times greater than that for the very angular fold (K being proportional to time), which might be expected inasmuch as the stress difference is very small. Fig.5B shows, in stages, the growth of the very angular fold from the circular arc. At 60° limb-dip the difference in dip between the segment at the inflection point and the penultimate one to the hinge is less than 2°. This is confirmed by inspection of the graph for R = 1.1 in Fig.3: for dips between 1° and 40° the function $f(\theta)$ hardly changes at all. Its

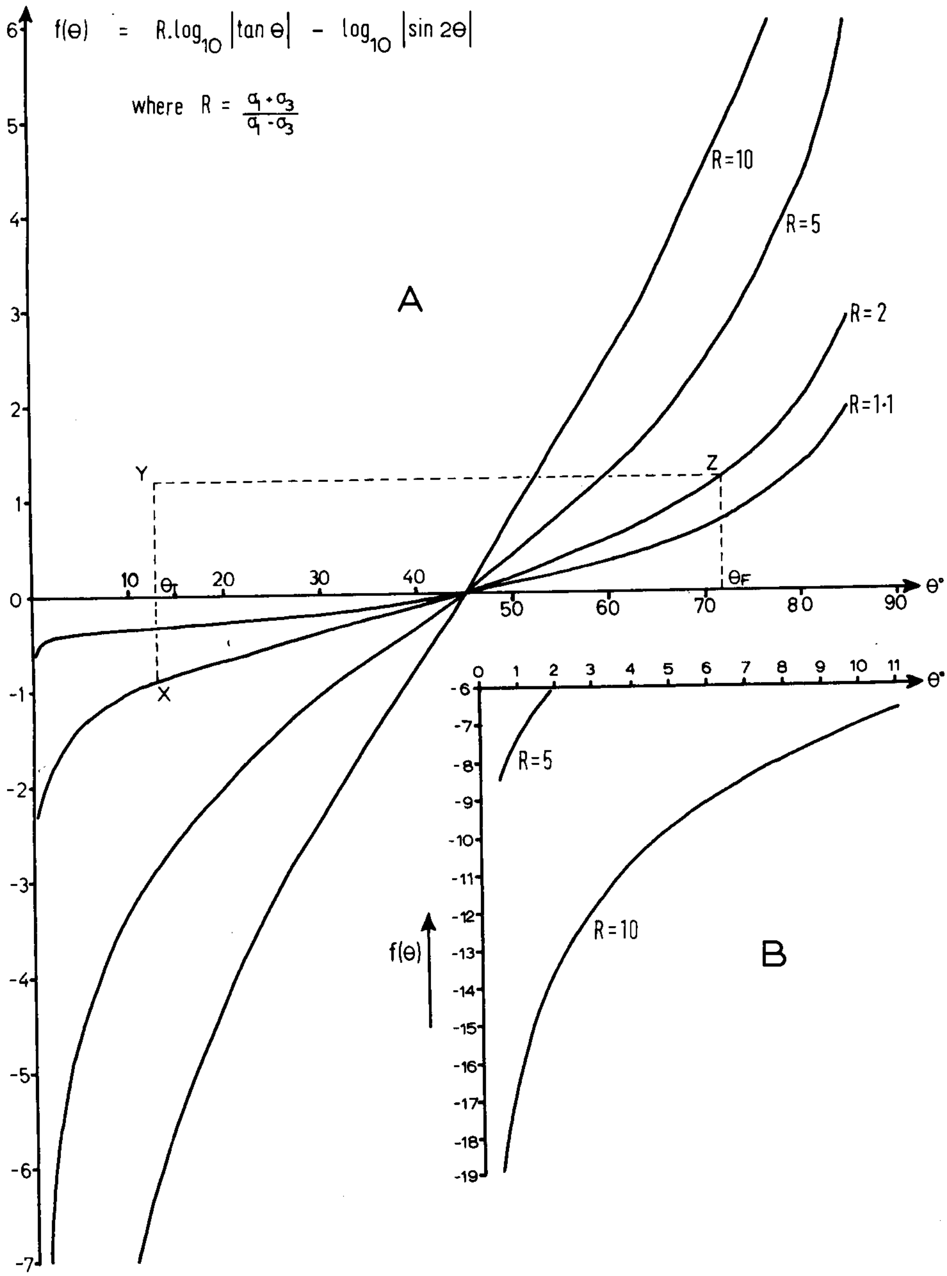

Fig.3. A. Family of curves of f(θ) against dip θ expressing the variation in dip after a given time of an element acted upon by stresses σ_1 and σ_3. An element dipping initially at θ_I, say, fixes a point X on one of the curves representing a fixed ratio of σ_1 to σ_3. A constant XY, proportional to time, is added to the value of the function f(θ), to define a point Z on the same curve, from which the final dip θ_F can be read. B. Extension of the graphs to small dips and large values of R.

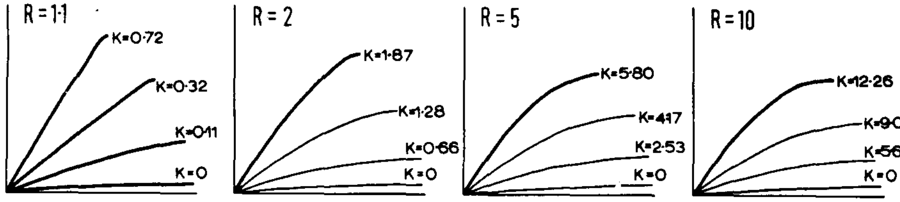

Fig.4. Reconstructions for different values of $R = (\sigma_1 + \sigma_3)/(\sigma_1 - \sigma_3)$ of the initial quarter-wavelength circular arc ($K = 0$) with limb-dip 5°. The three later curves in each case are for equal limb-dips of 20°, 40°, and 60°, not for equal times (proportional to K). The initial arc is divided into ten elements. The horizontal and vertical axes are horizontal and vertical distance, respectively, both to the same scale.

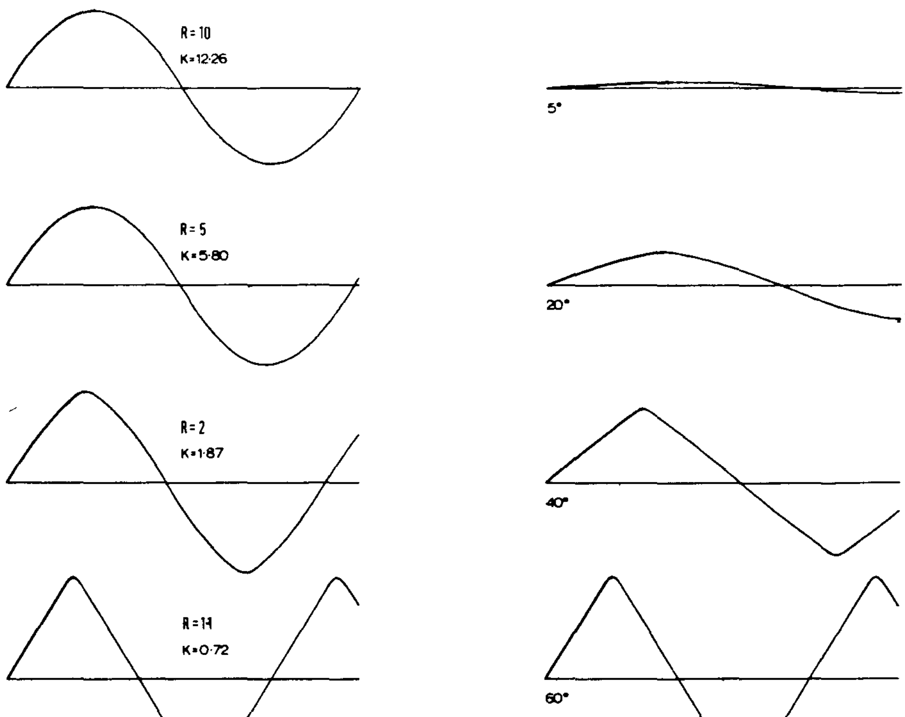

Fig.5. A. The quarter-wavelength curves of Fig.4 having 60° limb-dip, extended to a complete wavelength or more. B. Stages of development of the angular fold (R = 1.1) from the initial circular arc.

curious asymmetry is due to the importance, for small values of R, of the $\log|\sin 2\theta|$ term in the function. This term is a direct result of having $d\theta/dt$ inversely proportional to σ_n (see Appendix), which is the mathematical expression of the premise that viscosity is proportional to σ_n.

Preliminary trials show that the convexity of the starting curve or its shape, e.g., a sinusoidal wave instead of a circular arc, makes little difference to the final results. A circular arc with a limb-dip of 17° gives almost straight-limbed folds for $R = 1.1$, whereas sinusoidal starting curves tend to emphasize angularity. Division of the quarter-wavelength arc into only five elements instead of ten does not appreciably affect the shape of the reconstructions. A circular arc with a 5° limb-dip was chosen to obviate any initial bias towards angularity, and as the nearest practical approximation to unfolded layers.

An alternative model, mentioned above, is one in which viscosity η is both proportional to normal stress σ_n and inversely proportional to shearing stress τ. Following the same procedure as before, eq.2 is replaced by:

$$\eta = \sigma_n / \gamma . \tau \tag{7}$$

where γ is a new constant replacing c. The integral, equivalent to eq.4, resulting from this together with eq.1 is:

$$\left[\frac{1}{\sin 2\theta} - \frac{\sigma_1 + \sigma_3}{\sigma_1 - \sigma_3} \cdot \frac{1}{\tan 2\theta}\right]_{\theta_I}^{\theta_F} = (\sigma_1 - \sigma_3) \cdot \gamma \cdot T \tag{8}$$

The family of graphs of the function:

$$f(\theta) = \frac{1}{\sin 2\theta} - \frac{R}{\tan 2\theta} \tag{9}$$

plotted against θ for different values of R has remarkable similarity to Fig.3, the main difference being that the curves all pass through the point $(1, \pi/4)$ instead of $(0, \pi/4)$. Slightly smaller values of R are required for the more asymmetric curves, so it follows that a larger ratio of σ_1 to σ_3 will be needed to produce the same angular fold as in the previous model. However, the presence of the $(\sigma_1 - \sigma_3)$ term on the right-hand side of eq.8 means that such a fold will develop far more quickly than its equivalent in the earlier analysis.

DISCUSSION

Geological significance

In the deformation of initially gently folded well-bedded rocks by flexural flow, the theory predicts that upright angular folds will develop where the horizontal component of stress σ_1 is much larger than the vertical component σ_3. This accords with the observation that they tend to be found in the upper levels of orogenic belts, where the accompanying metamorphism is very low-grade. The gentle buckles required to start the process are bound to occur when unfolded horizontal beds are subjected to a plane-parallel regional compressive stress σ_1.

A common feature of large-scale angular folds found in such geological situations is their asymmetry, the limbs on one side of anticlinal hinges being consistently longer than on the other side. It has been suggested, for instance by Freshney et al. (1966) working in north Cornwall, England, that this asymmetry is due to the superimposition of a shear stress upon the orthogonal compressive stresses (Fig.6). Ramberg (1963b) and Ghosh (1966) have shown theoretically and experimentally that a combination of shear and compressive stresses parallel to initially plane layers is required for such "drag" folds to evolve. On the

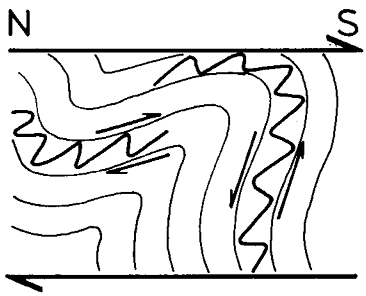

Fig.6. Interpretation of major geological structures in north Cornwall, England, as due to superimposed shear couples. (After Freshney et al., 1966.)

mesoscopic scale the shear stress is induced by "drag" on the limb of a major structure, whereas the inclination of the major folds themselves is due to a regional horizontal shear couple caused by tectonic transport of material.

However, in neither case can the superimposed shear stresses arise as soon as a presumed regional horizontal stress is applied, but will grow in the former case as the major fold develops and in the latter case as tectonic transport ensues in the development of the fold belt. The effect of adding a growing horizontal shear couple to a horizontal and vertical orthogonal compressive stress system is the same as rotating the orthogonal axes, and is shown in sketch form in Fig.7. So whereas a stress field acting on gently buckled beds inclined at a large angle to the maximum principal stress axis would only result in flattening and stretching of the beds, gradual rotation of this axis σ_1 out of an orientation initially parallel to the "planes" of the gently buckled beds will result in asymmetric folds. Unfortunately the problem is complicated by the continuous variation of the now rotating σ_1 and σ_3 as the shear couple S grows (see Fig.7B′ and 7C′).

Duration of the folding process

If the viscosity of the rocks is known it should be possible to estimate the time taken for a given fold to form. From eq.2:

$$c = \sigma_n/\eta \tag{10}$$

For a depth of burial of 4 km the dry-rock lithostatic pressure is of the order of 1 kbar. For initially unfolded rocks this is equal to σ_n (assuming no fluid pressure), which will therefore be about 10^9 dyn/cm^2.

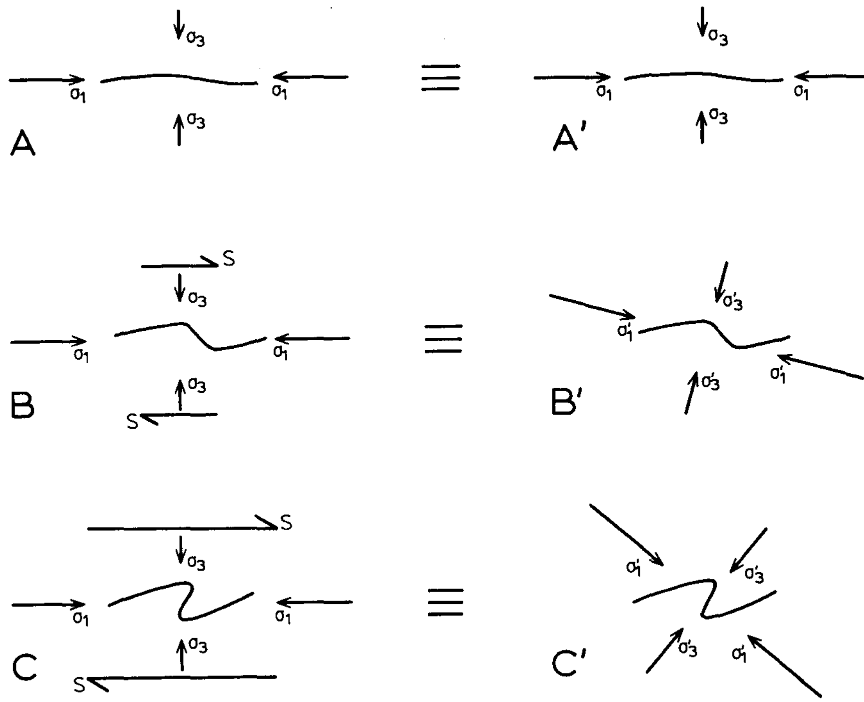

Fig.7. Schematic development of asymmetrical folds. A. Initial buckling of a horizontal layer under horizontal maximum compressive stress σ_1 and minimum compressive stress σ_3. B. Superimposition of small shear couple S parallel to σ_1, resulting in B′, with new maximum and minimum stresses σ_1' and σ_3', rotated slightly in the same sense as S, C and C′. Large shear couple S resulting in large rotation of orthogonal compressive stresses and development of an asymmetrical fold.

Assuming for the sake of argument a viscosity of 10^{21} poises—one order of magnitude less than that calculated for the upper mantle from the Fennoscandian uplift—then:

$$c = 10^9/10^{21} = 10^{-12}/\text{sec} = 3 \cdot 10^{-5}/\text{year}$$

But $2cT = K$ (from eq.6), and for the very angular fold of limb-dip 60°, $K = 0.7$; hence $T \sim 10^4$ year.

In the above example of angular folding at shallow depth the value of $R = 1.1$ implied that the horizontal stress σ_1 must have been about ten times greater than the vertical stress σ_3, i.e. $\sigma_1 \sim 10^{10}$ dyn/cm^2. Under the same conditions but at increased depth, say 30 km, σ_3 (due to depth of burial) would be about 10^{10} dyn/cm^2. Assuming that the same horizontal stress σ_1 is operating, the very rounded type of fold would develop here, as the now small difference between σ_1 and σ_3 implies a large value of R (N.B. σ_1 must of course be greater than σ_3 for folding to occur). For a Barrow's Zones type of regional metamorphism the vertical temperature gradient of about 25°C/km would mean a temperature of 600°C at 30 km, whereas at 4 km it would have been under 100°C. The well-established exponen-

tial decrease of viscosity with temperature in going from a depth of 4 km to 30 km would thus be about a factor of 10, but the corresponding increase with pressure probably cancels this out, to within an order of magnitude. So for $\sigma_n = 10^{10}$ and $\eta = 10^{21}$, $c = 10^{-11}$/sec $= 3 \cdot 10^{-4}$/year.
For the very rounded fold of limb-dip 60°, $K = 12$, so:

$$T = K/2c \sim 2 \cdot 10^4 \text{ year.}$$

The calculation above suggests that in the formation of an orogenic belt angular folds and more rounded folds of similar amplitudes will develop at shallower and greater depths, respectively, in similar time spans, assuming that a flexural flow mechanism can be applied over this range of temperature and pressure.

The actual magnitude of the time span depends upon this viscosity. While 10^4 year seems to be a reasonable order of magnitude, J.Hall (personal communication, 1970) points out that the value of 10^{22} poises from the Fennoscandian uplift has been derived by the reverse procedure from that used here, also assuming a simple theoretical model. Such a cyclic sequence:

time estimate → rheological model → viscosity estimate → rheological model → time estimate

must clearly be viewed with caution.

Previous work concerning angular folds

Patterson and Weiss (1966) and Weiss (1968) carried out experimental deformation of phyllite and multilayers of thin cards, giving kink bands which on migration and intersection result in chevron folds. Donath (1968a and b) also produced kink bands experimentally, and points out that these types of experiment involve a flexural slip mechanism, together with a sudden release of elastic strain energy in brittle deformation. Ramberg (1964) and Ghosh (1968), working with elastic and viscous models of rather thicker-layered multilayers, have produced chevron folds in the inner part of the multilayers, grading into rounded folds in the outer layers. The predominant mechanism here is probably flexural slip between the layers (especially in Ghosh's models with "great ease of sliding", in which there was a tendency for gaps to develop at the hinge with increasing deformation), although a certain amount of flexural flow must have occurred within each layer, since the folds are not ideally concentric. It is therefore apparent that for extended media or finite multilayers in which flexural slip is the predominant mechanism (together, perhaps, with elastic effects), angular folds result from the simple geometric requirement of maintaining constant layer thickness. Any variation of viscosity with stress – although probably present in most materials, including rocks – is of subsidiary importance. Such folds tend to have a short arc length relative to the thickness of the folded unit, and tend to vary in style and amplitude from one wavelength and layer to the next, either in an irregular fashion (the kink band type) or systematically (the buckling multilayer).

Bayly (1970) has produced experimental angular folds from a planar multilayer in which isotropic layers of a stitching wax-oil mixture alternated with anisotropic layers of aluminium flakes embedded in the mixture and orientated parallel to the layering. The constancy in style of the folds and the strong anisotropy of the layering suggest, firstly, that a flexural flow mechanism operated, as required by the present theory. Secondly, the requirement of viscosity increasing with normal stress was very probably fulfilled, for although the stitching wax is quoted as having a viscosity which "varies little over a wide range in stress magnitude", it was mixed with oil, whose viscosity increases exponentially with pressure (see, for instance, Bradbury et al., 1951). Therefore the matrix mixture of 90% wax and 10% oil probably had a viscosity which increased somewhat less than exponentially with pressure. Thirdly, the prediction from the present theory that angular folds develop when σ_1 is much bigger than σ_3 seems to be obeyed, because compressive stress σ_1 was applied by a screw mechanism parallel to the layering, while the only stress normal to the layering, apparently, was atmospheric pressure (σ_3). The box-shaped model was confined parallel to σ_1 and at right angles to the layers by lubricated rigid boards, which would therefore have exerted an intermediate stress σ_2 normal to σ_1 and σ_3.

Angular folds are most commonly likened to kink bands, because of their most obvious features – strong foliation or bedding, and strain by gliding of these planes over one another. However, the only rigorous theoretical treatment of angular folds as brittle structures rather than as viscous or plastic phenomena has been by Price (1967). Having previously argued for the consideration of competent rocks in the upper layers of the crust as a solid and not as a viscous fluid (Price, 1966), he applies the concept of a "long term" finite yield strength to a single competent layer. On brittle failure symmetric and asymmetric angular folds result; but the model may not have more than a limited application to geological structures because:

(1) All folds so produced are required to have a fracture plane coincident with the hinge plane, which is frequently not the case (Fig.8A).

(2) Deformation by brittle failure makes angular folds a class distinct from all other types of folds, whereas in practice every gradation from angular to rounded profiles can be found.

(3) The theory applies only to a single competent unit, which, as Chapple observes, is a geologically unrealistic situation for the occurrence of angular folds. The generalisation by Price of his theory to a multilayer is not made analytically, but on an appeal to "field evidence".

(4) The concept of strain symmetry reflecting stress orientation, maintained by other theoretical fold models, is not obeyed by the extension of the model to asymmetrical buckles (Fig.8B).

The concept of a "long term" yield strength arose from experiments of up to 100 days duration on rocks, suggesting that the "long term" strength varied from 20 – 60% of the "instantaneous" yield strength. But Robertson (1955) has pointed out that, theoretically, for a finite stress difference, creep strain is zero only at absolute zero of temperature.

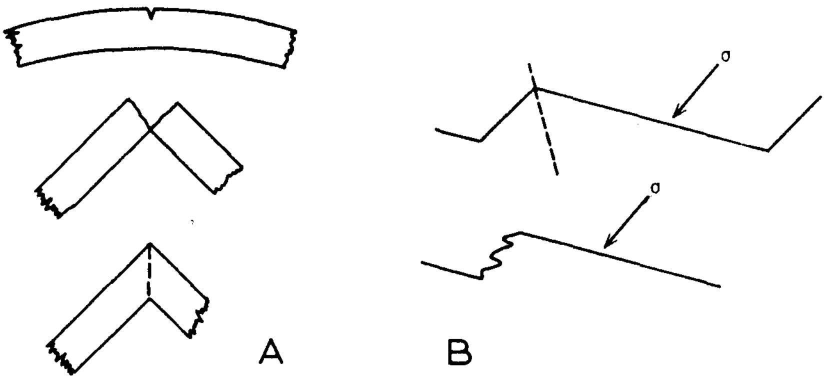

Fig.8. A. Development of an angular fold by brittle failure of a single layer with a finite yield strength. B. Development of an asymmetrical fold under inclined maximum compressive stress σ. The fold axial plane (shown dotted) is not normal to σ, and the parasitic folds on the shorter limb have an orientation unrelated either to the major fold or to σ. (After Price, 1967.)

Donath and Fruth (1971) have carried out extensive work on the effects of different strain rates on the strength and deformation mechanisms of four representative rock types, and by extrapolation conclude that rocks which deform by cataclastic flow may have a significant strength even at geological strain rates of $10^{-13} - 10^{-14}$/sec. However, it should be noted that their definition of strength means "sustained differential stress" under deformation at constant strain rates, whereas they refer to the elastic limit of strength as the "yield stress". Thus they define two strengths: (a) the maximum stress under continuous deformation at a given strain rate "that the material is capable of withstanding" (Donath and Fruth, 1971, p.351); and (b) the minimum or yield stress below which no permanent strain can occur. Their conclusions refer to the former strength, and do not invalidate the possibility that the yield stress may tend to zero over long periods. For a discussion of the relative merits of constant strain-rate and creep (constant stress) experiments, see Heard, 1963. So it may well be that for periods of, say, $10^2 - 10^6$ years (from the estimates of time above), "yield stress" or "yield strength" becomes negligibly small, and rocks, even in the shallower levels of the crust, can be treated to a first approximation as viscous fluids.

CONCLUSION

A range of fold profiles varying in style from angular to rounded can be generated from a gentle buckle by means of a flexural flow mechanism, assuming two simple premises: (1) that shear strain rate is proportional to shear stress (the definition of a Newtonian fluid); and (2) that the constant of proportionality (i.e. the viscosity) associated with this is itself proportional to the normal compressive stress.

The mathematical expression of these statements, on integration, gives a variably asymmetric function – the family of curves of Fig.3 – and it is the variation of this function which controls the variation in fold style. When the horizontal stress σ_1 is much greater than the vertical stress σ_3 (i.e., R small), as might occur in the upper levels of the crust under orogenic conditions, angular folds are generated from initial buckles in a medium with strong planar anisotropy, such as a well-bedded sequence of strata. When σ_1 is only just greater than σ_3 (i.e., large values of R) a more rounded style develops. However, if other conditions are equal, it takes far longer for the latter style to grow to a given amplitude than it does for the angular fold to reach the same amplitude; this result is to be expected because the rate of folding is controlled by stress differences rather than by stress magnitudes. In addition, the rate at which a given fold develops is directly proportional to the viscosity of the rocks, i.e. the "static" viscosity measured under standard conditions of temperature and pressure. As has been shown, this viscosity increases with normal compressive stress, and is therefore increased by varying amounts over a fold profile during the folding process. "Absolute" estimates of the duration of such a process thus depend upon measurements of rock viscosities, which are not yet known even to within an order of magnitude.

Nevertheless, both premises are probably too idealised for wide application to rocks, because other factors such as the influence of bedding thickness have been omitted. The model is intended only as an introduction to the probable importance of stress on viscosity, which up till now has apparently been neglected. However, the results will hold qualitatively as long as viscosity increases with compressive stress (whether proportionately, or exponentially, or in some other way), and either does not change or else decreases with shearing stress.

Work is in hand to develop a function describing a whole quarter-wave arc, not just an element of it as at present. The function could then be expressed as a Fourier half-range series and related to the simplified fold description outlined by Stabler (1968). The four simple field measurements on a fold profile required by this method, together with deductions from the degree of metamorphism of the rock, would then yield estimates of the absolute stress magnitudes and (if the viscosity were known) the time duration of the folding process.

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APPENDIX

$$\frac{d\theta}{dt} = \frac{c\tau}{\sigma_n}$$

$$\tau = \frac{\sigma_1 - \sigma_3}{2} \cdot \sin 2\theta$$

$$\sigma_n = \frac{(\sigma_1 + \sigma_3)}{2} - \frac{(\sigma_1 - \sigma_3)}{2} \cdot \cos 2\theta$$

$$\therefore \frac{d\theta}{dt} = \frac{c \cdot \frac{(\sigma_1 - \sigma_3)}{2} \cdot \sin 2\theta}{\frac{(\sigma_1 + \sigma_3)}{2} - \frac{(\sigma_1 - \sigma_3)}{2} \cdot \cos 2\theta}$$

Let $\frac{(\sigma_1 + \sigma_3)}{2} = \overline{\sigma}$ and $\frac{(\sigma_1 - \sigma_3)}{2} = \Delta\sigma$

$$\therefore \frac{\overline{\sigma} - \Delta\sigma \cdot \cos 2\theta}{c \cdot \Delta\sigma \cdot \sin 2\theta} \cdot d\theta = dt$$

$$\therefore \frac{\overline{\sigma}}{\Delta\sigma}\int \frac{d\theta}{\sin 2\theta} - \int \frac{\cos 2\theta}{\sin 2\theta} \cdot d\theta = c\int dt$$

Let $2\theta = \alpha$. $\therefore 2\,d\theta = d\alpha$

$$\therefore \frac{\overline{\sigma}}{\Delta\sigma}\int \frac{d\alpha}{\sin\alpha} - \int \frac{\cos\alpha}{\sin\alpha} \cdot d\alpha = 2c\int dt$$

(See, for example, Dwight (1961) p.96 and p.110 for the standard integrals.)

$$\left[\frac{(\sigma_1 + \sigma_3)}{(\sigma_1 - \sigma_3)} \cdot \log|\tan\theta| - \log|\sin 2\theta|\right]_{\sigma_I}^{\sigma_F} = 2cT$$

where $T = t_F - t_I$.

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